Limitations in the Back-Analysis of Strength from Failures

Rick Deschamps[1] and Greg Yankey[2],
Nicholson Construction Company, 12 McClane Road, Cuddy, PA
FMSM Engineers, 1409 North Forbes Road, Lexington KY

ABSTRACT

Stability failures are often "back analyzed" in an attempt to estimate the operative shear strength. In fact, back-analysis is commonly believed to be one of the most reliable ways to estimate soil and/or rock strength. However, this paper provides examples to illustrate specific situations in which back-analysis of failures can lead to misinterpretation of strength. Examples from earth and concrete gravity dams are used, and consideration is given to both 2-D and 3-D idealizations. The cases demonstrate that interpreted strength can be in significant error, and in practically all cases the errors are unconservative. Finally, this paper illustrates that back-analysis is reliable only when the model and all assumptions are reasonable and accurate representations of the real system.

INTRODUCTION

Back-analysis is an approach commonly used in geotechnical engineering to estimate operable material parameters in-situ. This approach is popular because there are significant limitations in the use of laboratory and in-situ test results to accurately characterize a soil profile. Numerous studies have demonstrated the use of back-analysis techniques for the determination of soil parameters. There are also several publications that describe limitations of back analyses (Leroueil and Tavenas, 1981; Azzouz et.al., 1981; Leonards, 1982; Duncan and Stark, 1992; Gilbert et. al. 1998; Tang, et. al. 1998; Stark et. al. 1998). Although these publications describe many of the pitfalls of back-analyses, it is the authors' experience that these facts are not fully appreciated by many practicing engineers. Accordingly, the objective of this paper is to illustrate how different assumptions made in the back-analysis of failures can influence the interpreted shear strength. A recognized fact that must be recognized is that a conservative design assumption is unconservative when used in back-analysis because other values of shear strength may be conservatively in design causing the back calculated strength to be over estimated. As engineers, we commonly build some conservatism into the selection of design parameters. This tendency, often subconscious, leads to unconservative interpretations of strength in using back-analysis. Moreover, the models that are typically employed in geotechnical engineering embody conservative assumptions. Accordingly, the models can also lead to unconservative results when used in back-analyses, and therefore, must be factored into interpretations. What is conservative in design is unconservative in back-analysis, and vice versa. In this paper, some of the factors that complicate the use of back-analysis are described first. Subsequently, specific project examples are used to illustrate the magnitudes of potential errors.

FACTORS THAT INFLUENCE INTERPRETED SHEAR STRENGTH DURING BACK-ANALYSIS.

1. The relative strength of materials in heterogeneous profiles impacts interpretations of the target material strength. Often it is desirable to back-calculate the strength of a weak layer or seam. However, to accurately back-calculate the strength of the desired material, the strength of all other materials must be known.

2. The slip surface analyzed must be the same as the actual rupture surface to effectively back-calculate the strength of a deposit. Leonards (1982) describes several cases where a back-analysis was reported to show a safety factor near 1.0 using laboratory or in-situ determined shear strengths, but the failure surface in the analysis is not consistent with the actual rupture surface. Conclusions are sometimes drawn that the limit equilibrium method used with shear strengths obtained by some specific approach can predict the failure, but not the surface (e.g. Skempton 1945). This logic is flawed because the actual slip surface will demonstrate different back-calculated strengths than the "critical" surface obtained from analyses

3. . Furthermore, a significant challenge is that the actual rupture surface may be known at only a few locations, if any.

   Another example of uncertainty in the slip surface is the presence of a tension crack, its depth, and whether it is full of water. For materials that are characterized as having relatively high cohesive strength, the assumption of a tension crack will have a significant influence on interpreted stability, and therefore, the back-calculated strength.

4. Knowledge of the pore water pressure is required to determine effective stresses, and therefore strength. Sometimes there is pre-failure piezometric data at select locations, sometimes measurements are made post failure, and other times pore pressures are estimated. However, it must be recognized that the actual distribution of water pressure can be complicated, and that the operable pore pressures at failure, including shear-induced pore pressures, cannot be reliably measured.

5. Practically all slopes have a three dimensional component. Neglecting this component in back-analysis will lead to an overestimation of strength. However, “end effects” are not easily accounted for because the influence on stability can vary over a broad range. Azzouz et. al, (1981); and Stark and Eid, (1998) provide estimates of the influence of three-dimensional factors in stability analyses. In general, for soil slopes and embankments the end effects appear to increase stability by from 5 to 30 percent.

6. Progressive failure in strain softening materials will also affect interpretation of strength. If the back-calculated strength is to be used for similar slopes in similar stratigraphy, the back-calculated operable strength may be useful (Duncan and Stark, 1992). However, if the loading condition induces a significantly different stress path, the back-calculated strength from another stress path or geometry may be misleading. For example, using strength back-calculated from slope failures may not be appropriate in foundation design.

7. Mohr-Coulomb strength is defined by a friction angle and a cohesive intercept. Determination of these parameters individually is typically not possible unless significant redundant data is available (Duncan and Stark, 1992).

PROBLEM ILLUSTRATED

Relative safety is typically quantified by use of a safety factor that is defined as some ratio of resisting forces (and/or moments), to the forces required for equilibrium (driving forces). This relationship can be idealized for the case of general wedge surface for a slope using Eqn 1, wherein the numerator is related to the strength and the denominator to the mobilized stresses.

$$SF = \frac{\sum \{c_i + (\sigma_i - u_i)\tan\phi_i\}\ell_i}{\sum \tau_i \ell_i} \quad (1)$$

Where:

c	=	cohesive intercept
σ	=	normal stress
u	=	water pressure
ϕ	=	friction angle
τ	=	mobilized shear stress
ℓ	=	length of rupture surface in layer
i	=	wedge being considered

Consider Eqn 1, if the strength parameters (c and ϕ) are underestimated, the safety factor is reduced, and therefore, the analysis is conservative. Likewise, if the water pressure is overestimated, the computed safety factor is also conservative. Furthermore, if there is resistance from three-dimensional effects that is not accounted for in the numerator of Eqn 1, the safety factor is underestimated, so the results are conservative. As engineers, we often take solace in these "hidden" conservatisms such that we tend to lean to the conservative side of data containing scatter.

In the back-analysis of a failure, the assumption is made that the safety factor is 1.0 so that resisting forces equal the driving forces. For the simple case of a two-wedge system with no cohesion, Eqn 1 becomes:

$$\{(\sigma_1 - u_1)\tan\phi_1\}\ell_1 + \{(\sigma_2 - u_2)\tan\phi_2\}\ell_2 = \tau_1\ell_1 + \tau_2\ell_2 \quad (2)$$

The right side of Eqn 2 is dictated by equilibrium and can be considered "known" for a given geometry and rupture surface. The objective of back-analysis is to determine the strength components on the left side of Eqn 2. Equating the resisting forces with the driving forces, by fixing the safety factor at 1.0, leads to the condition that conservative design assumptions are unconservative in back-analysis, as is illustrated by the following scenarios.

- Assume that it is desirable to back-calculate the strength of a weak material (Layer 2) from a failure in which there is high confidence in both the failure surface location and the water pressure. Given that the right side of Eqn 2, and σ_1 and σ_2 are obtained from equilibrium, an estimate of ϕ_1 is required in order to calculate ϕ_2. Note that because of the imposed equality, the magnitude of ϕ_2 must increase as the magnitude of ϕ_1 decreases. Therefore, if ϕ_1 is underestimated (typically a conservative assumption), ϕ_2 will be overestimated (an unconservative result).
- Similarly, overestimating the water pressure (typically conservative) will reduce the normal effective stress and lead to a larger (unconservative) back-calculated shear strength in order to satisfy Eqn 2.
- Virtually all slope failures possess a three dimensional aspect that is commonly neglected. To appropriately describe this condition, another resistance term should be included on the left side of Eqn 2. However, neglecting the three-dimensional resistance (normally a

conservative assumption) leads to higher back-calculated strengths (unconservative result) in order to satisfy the equality.

These are just a few simple examples that illustrate a general point: assumptions that are conservative in design are unconservative in back analysis. Importantly, as engineers we are generally conservative by nature in both our models and in our parameters selection. Accordingly, we must resist this tendency or risk unconservative consequences when performing back-analysis.

QUANTITATIVE EXAMPLES OF POTENTIAL ERRORS IN BACK-ANALYSIS

Project examples are provided to illustrate the challenge and potential errors that can be present in back-analysis for Items 1 through 4 described in the previous section. Succinct examples could not be developed for Items 5 and 6, however, their impact is generally present. The examples are drawn from three case histories. One of the case histories is of Grandview Lake Dam located in Bartholomew County, central Indiana. The dam is a privately owned homogeneous earth dam that was built to form a recreational lake and waterfront homesites. This case history is used to illustrate the dependence that the back-calculated strength along a weak seam has on assumptions of strength in other zones. The second case history is a stability assessment/design of a very large (2.0 million m^3) spoil pile adjacent to the Kanawha River at Marmet Lock and Dam, West Virginia. This project demonstrates the importance of characterizing the actual rupture surface. The third case history is Lock and Dam 10 on the Kentucky River. This is a relatively small concrete gravity dam owned by the Commonwealth of Kentucky and built circa 1905. This project demonstrates the importance of understanding the pore pressure distribution (uplift pressures) along a potential slip surface and three-dimensional effects when back-calculating strengths.

Material Strengths. Thirty years after construction Grandview Lake Dam began sliding downstream at rates of approximately one centimeter per day along a distinct thin, planer weak seam in the claystone foundation (See Deschamps et. al. 1999, for a more complete description of the project). Emergency berms were placed to slow movements and back-analysis was viewed as an important tool for estimating the strength along the planer slip surface. A cross section of the dam at the time of failure, the location of the assumed rupture surface developed from inclinometer measurements, and the location of the piezometric surface obtained from piezometers is shown in Figure 1. The dam was constructed primarily of glacial till and residual soils weathered from claystones. The first challenge was to select the operable strength of the dam materials. There was no distinct zonation of materials in the dam, and therefore, no basis for subdividing the dam into discrete materials. Figure 2 illustrates the results of available consolidated undrained (CU) triaxial tests. These tests included compacted specimens (dashed lines) and undisturbed samples obtained from Shelby tubes. Most tests were completed in 1986 (Labs A and B), while the tests performed in 1997 (Lab C) were conducted on the compacted residual mudstone. Given the heterogeneous nature of the materials in the dam it was viewed as unlikely that additional tests would further clarify the operable strength without knowledge of material quantities and zonation. Moreover, there was little time for detailed exploration and testing given the emergency nature of the project.

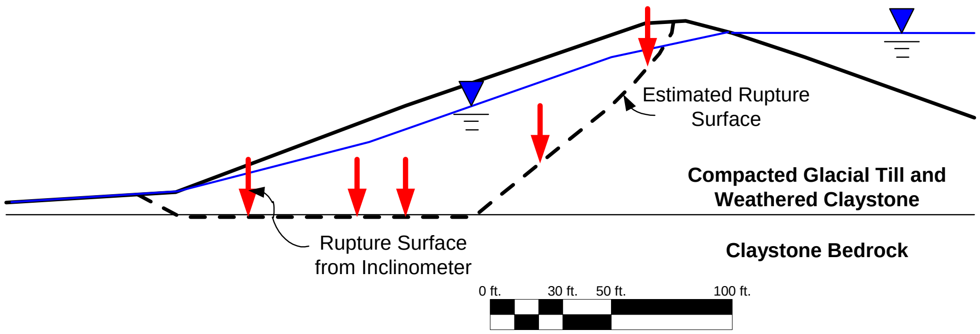

Figure 1. Cross-Section through Grandview Lake Dam with Piezometric Surface, Inclinometer Locations and Assumed Rupture Surface.

Five different characterizations of embankment strength are considered here for back-analysis purposes to characterize the weak seam strength. A lower bound, average, and two upper-bound interpretations are shown in Figure 2.. Two upper bound strengths are used here because one represents a higher cohesive intercept while the other a higher friction angle. Given these four characterizations of dam strength and the conditions of Figure 1, a summary of the back-calculated friction angles in the weak seam is shown in Table 1. The strength along the weak seam was characterized as having a zero cohesive intercept because it was rationalized that this material was at or near its residual strength because the deformations along the very thin seam were significant, at least several inches. Moreover, there is little tendency for volume change at residual conditions such that shear induced pore pressure changes were considered negligible.

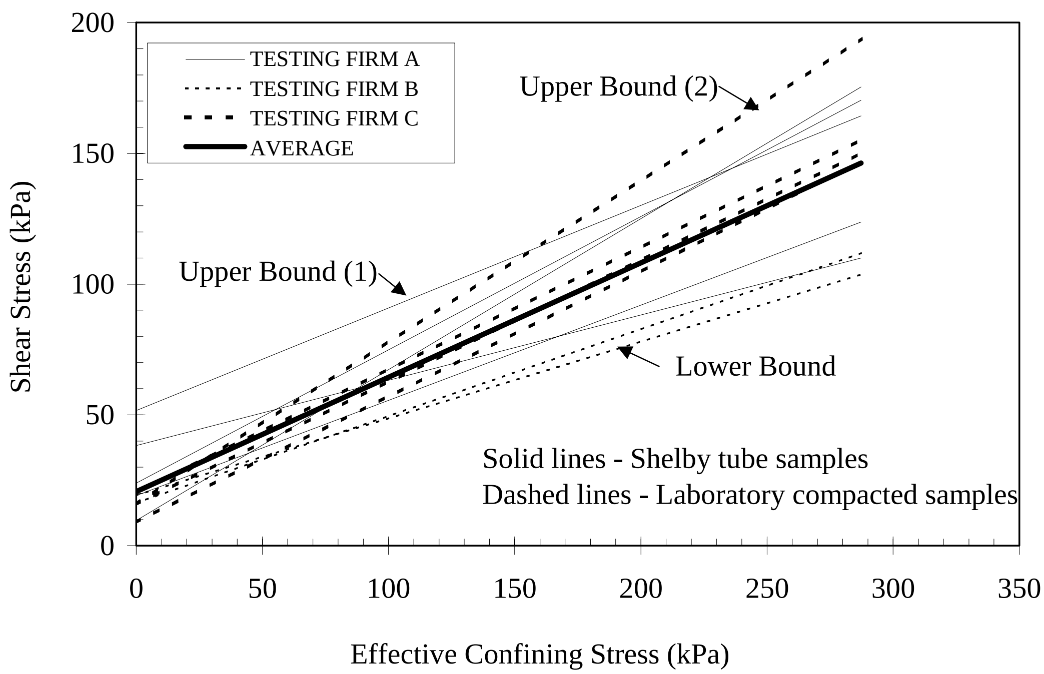

Figure 2. Consolidated Undrained Triaxial Test Data for Grandview Lake Dam

Table 1. Back-calculated strength.

Embankment Strength	Back Calculated Friction Angle (degrees)
Lower Bound	22-24
Upper Bound (High Friction Angle)	16
Upper Bound (High Cohesion)	11
Average	18

Table 1 illustrates the range in back-calculated friction angles is from 11 to 24 degrees, with the average strength providing 18 degrees. Although this range can be viewed as extreme, it is apparent that even if a narrower range of strengths were used to characterize the dam, the back calculated strength would still vary over an appreciable range. A conservative (low) estimate of embankment strength leads to a relatively high interpretation of strength along the weak seam. Note also the significant difference in back-calculated strengths for the two upper bound cases, wherein the case with primarily cohesive strength leads to a much lower back-calculated friction angle for the geometry considered because of the higher shear strength of the compacted materials.

Rupture Surface. Leonards (1982) discusses the importance of accurate characterization of the slip surfaces in back-analysis. The example that is used here is taken from analyses completed recently for a staged construction stability assessment of a spoil pile on a foundation containing loose sands and soft clays adjacent to a guidewall at Marmet Lock. The criterion for end of construction safety factor was 1.1. The end-of-construction design strength of the spoil material was specified as 77 kPa (1600 psf). Based on the results of limit equilibrium analyses using circular failure surfaces, this strength was believed to be adequate to meet the stability criterion for failure within the spoil material. However, during numerical modeling using the program FLAC developed by ITASCA (2002) to assess stability within the foundation, an interesting failure surface developed. Figure 3 shows a simplified cross-section of the system analyzed and the material properties used. The simulation included modeling the placement of the embankment in one foot "lifts," with failure occurring near the point of reaching full embankment height The apparent critical circular surface obtained from limit equilibrium using the program PC-STABL is shown on this figure and has a safety factor of 1.12. Also shown on the figure with shear strain contours is the failure surface obtained from numerical modeling, wherein the base of the rupture surface is located within the embankment and then is redirected to the foundation immediately below the embankment when approaching the toe of the slope. In hindsight, this seems logical because the lower vertical stresses near the toe leads to lower shear strengths in the frictional foundation than the assumed cohesive strength of the embankment in this region. However, this fact was not apparent initially and would not have been uncovered using limit equilibrium methods unless anticipated and rigorously investigated. The system that appeared to have a safety factor of 1.12, was in fact at a state of imminent failure. If the failure had occurred and was back analyzed by the conventional approach, not appreciating the subtleties of the true rupture surface, the strength would be found to be 63 kPa (1300 psf), or 81 percent of the actual strength. In this case, the design assumption was unconservative because the critical slip surface was not located. The unconservative design assumption leads to a conservative interpretation of strength in back analysis. Importantly, a limit equilibrium analysis of the surface predicted by numerical modeling leads to a safety factor of 1.0, provided the specific surface is evaluated or a non–circular search routine is adequately constrained to identify the surface.

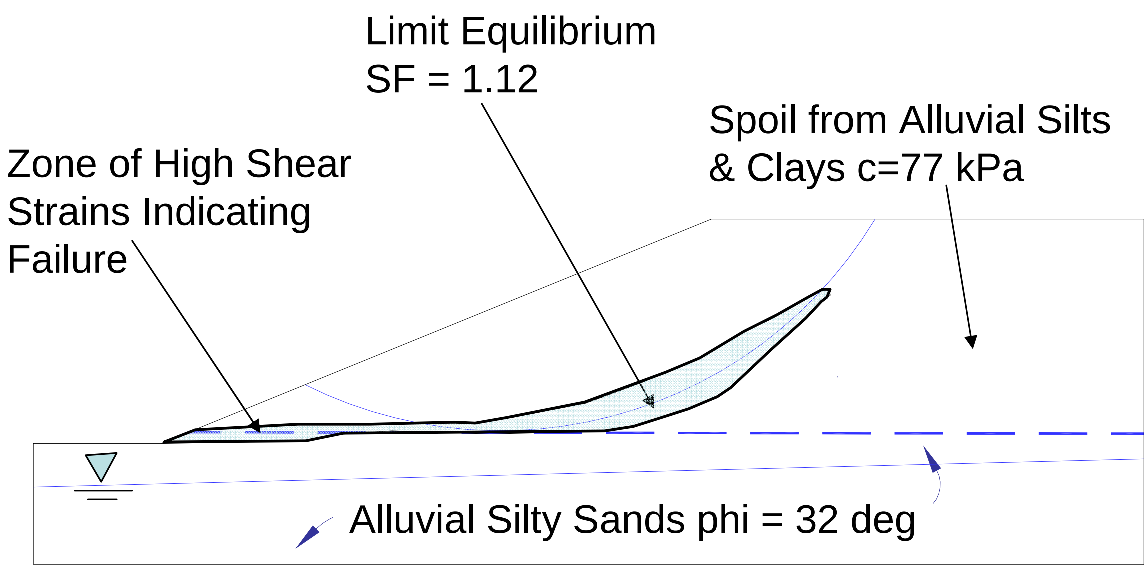

Figure 3. Comparison of Failure Surface by Numerical Modeling and Limit Equilibrium Circular Search.

Pore Pressures. Because pore pressures control effective stresses, they impact the interpreted back-calculated strength. A cross-section of Dam No. 10 on the Kentucky River is shown in Figure 4a. At the initiation of the remediation for this project there was no subsurface information for the site, and no original design information. It was known that the structure was founded on limestone, and that uplift pressures were not considered for design of dams founded on rock in this era (circa 1905). Previous technical reviews of stability assigned friction angles in the neighborhood of 45 degrees for the bedrock-concrete interface strength. A detailed subsurface investigation for the structure showed the limestone below the dam to be highly fractured. Moreover, extensive thin layers of shale, that had weathered to a clay consistency, were present at depths of one to two feet below the base of the dam. Laboratory test results showed that the effective stress friction angle of this clay was on the order of 27 to 30 degrees.

A conventional assessment of the stability of the dam indicates that a friction angle of 43 degrees is required to maintain stability for the maximum design flood, an event that occurs every year on average because of the headwater tailwater relationships in this riverine environment. Based on our interpretation of the foundation strength, we would have calculated a safety factor less than 1.0 for a common loading condition. Given the inconsistency between the required shear strength and our interpretation of subsurface conditions, we investigated potential limitations in the analyses. The first thing considered was the assumed pore pressure distribution. The complete length of dam is a spillway section and difficult access made the installation of piezometers too costly during the exploration program. Accordingly, the conventional assumption of a triangular distribution of head loss from heal to toe, with no head loss in the upstream sediments, was initially assumed, as it was in all previous analyses. These assumptions are shown in Figure 4b. However, given the fractured nature of the bedrock and the significant depth of sediment upstream, it is plausible that the majority of head loss is occurring in the sediment (Reduced pressures in Fig. 4). If this is the case, the required interface strength drops from 43 to 27 degrees, a substantial range for the variation in possible assumptions of water pressure. Although the possibility of reduced pore pressure may help explain why the dam could be stable given the weak clay seams, the three dimensional effects have a much greater influence, which is discussed in the next section.

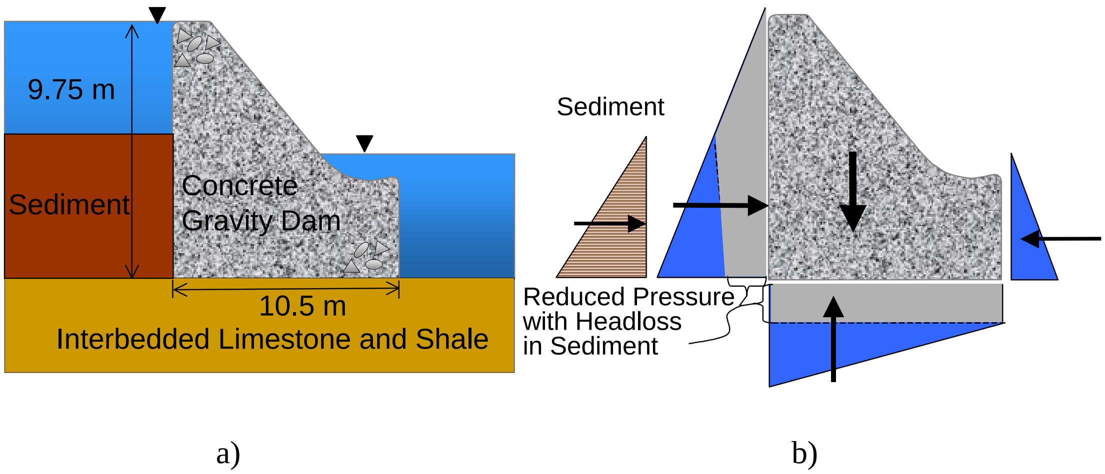

Figure 4. Cross-Section through Kentucky River Dam No. 10.
a) Idealized Section, b) Model for Stability Assessment.

Three-Dimensional or "End Effects."

Stability analyses are conventionally performed on idealized two-dimensional cross-sections, which are based on plane strain conditions. At Lock and Dam 10, coring through the dam indicated that the construction joints between concrete monoliths are essentially rubble, and could not be relied upon as shear connections between monoliths. Although it was considered imprudent to rely on the shear resistance between monoliths as a design consideration, it was recognized that some resistance was likely to be present. An attempt was made to estimate the magnitude of this resistance in order to understand the inconsistency between required strength and interpreted strength. Accordingly, numerical modeling with the program FLAC (ITASCA 2002) was used to model the entire dam as a beam. The dam is a spillway over its complete length of 240 feet; it has a height of 34 feet; a width of 32 feet; and is made up of ten monoliths 24 feet long. The assumption was made that only a nominal frictional resistance (35 degrees) was available between monoliths (no tensile or cohesive strength) and that the ends were fixed at the ends. The modeling effort produced a surprising result in which a zone within the dam formed a compressive arch that developed significant flexural resistance. Figure 5 illustrates the idealized model of the dam taken in plan view. The dam is attached to an abutment and training wall on the left, and the lock river wall on the right, both assumed to be stable. The distributed load on the beam was progressively increased to represent increasing the net hydrostatic pressures from higher pools. Based on this analysis, the compressive arch that develops has sufficient capacity to carry the complete hydraulic load acting on the dam during the maximum design flood, independent of any frictional resistance at the base, and with only frictional resistance between monoliths. This example clearly illustrates how difficult it would be to back-calculate strengths if there is a significant, but uncertain, three-dimensional influence. Although, the three dimensional effects are extreme in this case, influences of 5 to 30 percent are believed to be expected Azzouz et. al, (1981).

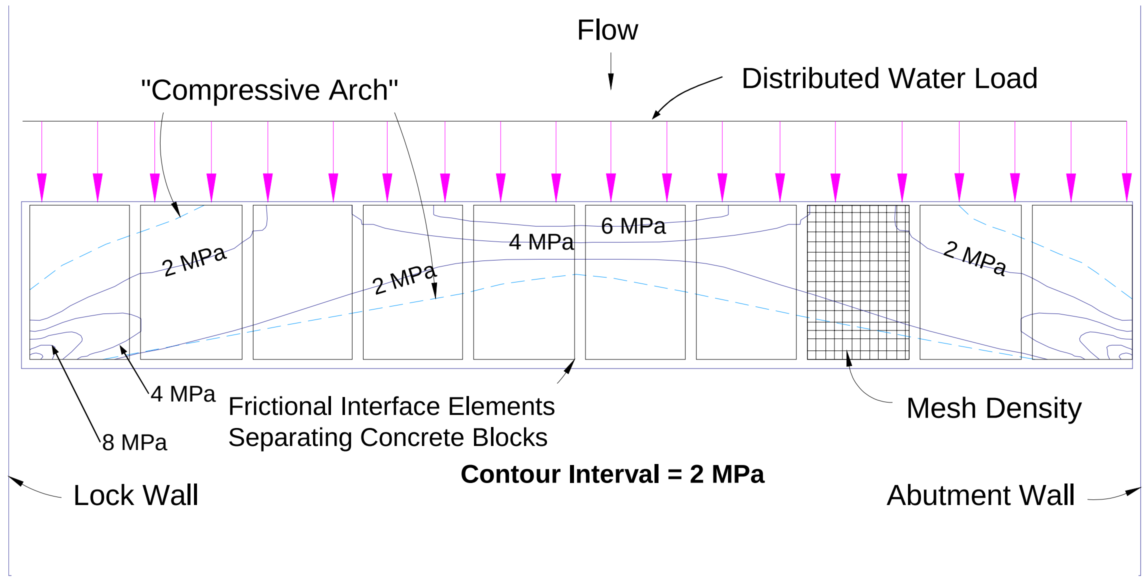

Figure 5. Plan View of Kentucky River Dam No. 10, Modeled as a Beam.

SUMMARY

Both a review of the definition of safety factor and specific examples are used to illustrate the significant challenge in accurately back-calculating shear strength from failures. The examples provided herein are intended to illustrate that successful back-calculation requires accurate information regarding geometry, material properties, and pore pressure distribution. In addition, adjustments must be made to account for the inherent limitations and assumptions of the models used in the analyses. All of these factors contribute to the results from back-analysis being at best approximate, and at worst in gross error. Finally, it is important to remember that all assumptions that are conservative in design are unconservative in back analysis.

REFERENCES

Azzouz, A.S., Baligh, M.M., and Ladd, C.C. (1981). "Three-Dimensional Stability Analyses of Four Embankment Failures," X Int. Conf. Soil Mech. Found. Engrg, Vol 3, pp 343-346, Stockholm.

Deschamps, R., Hynes, C., and Wigh, R. (1999). "Analysis and Stabilization of a Failing Earth Dam Founded on Claystone," ASCE, J. Performance of Constructed Facilities, Vol. 13, No. 4, pp. 143-151.

Duncan and Stark, (1992). "Soil Strengths from Back-Analysis of Slope Failures," Proc. Stability and Performance of Slopes and Embankments II, ASCE, GSP 31, pp. 890-904, Berkeley.

Gilbert, R.B., Wright, S.G., and Liedtke, E. (1998). "Uncertainty in Back Analysis of Slopes: Kettleman Hills Case history," J Geotech. Geoenviron. Engrg, ASCE, 124(12), pp1167-1176.

ITASCA (2002) "FLAC Users Manual," Versions 3.4 and 4.0.

Leonards, G.A. (1982). "Investigation of Failures," J. Geotech. Engrg, ASCE, 108(2), 187-246.

Leroueil, S. and Tavenas, F. (1981). "Pitfalls of Back-Analysis," X Int. Conf. Of Soil Mech. and Foundation Enginnering, Vol 1, pp 185-190, Stockholm.

Stark, T.D. and Eid, H.T. (1998). "Performance of Three-Dimensional Slope Stability Methods in Practice," J. Geotech. Geoneviron. Engrg., ASCE, 124(11), pp. 1049-1060.

Tang, W.H., Stark, T.D., and Angulo, M. (1998). "Reliability in Back Analysis of Slope Failures," Soils and Foundations, 39(5), pp73-80.